

Coherent Destruction of Tunneling and Dark Floquet State

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We study a system of three coherently coupled states, where one state is shifted periodically against the other two. We discover such a system possesses a dark Floquet state at zero quasi-energy and always with negligible population at the intermediate state. This dark Floquet state manifests itself dynamically in terms of the suppression of inter-state tunneling, a phenomenon known as coherent destruction of tunneling. We suggest to call it dark coherent destruction of tunneling (DCDT). At high frequency limit for the periodic driving, this Floquet state reduces to the well-known dark state widely used for STIRAP. Our results can be generalized to systems with more states and can be verified with easily implemented experiments within current technologies.

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Two-state and three-state models are the simplest quantum systems. Despite their simplicity, they often provide very good approximations to describe realistic physical systems and are capable of revealing a variety of fascinating quantum effects. The understanding of their ubiquitous features are nowadays being exploited for the manipulation and control of quantum states of small systems involving single atoms, photons, or nano-devices [1–3]. Coherent destruction of tunneling (two-state models) and dark state (three-state models) are two of the elegant prototype examples where deep understanding gained from quantum coherent effects in these simple systems are impacting the development of quantum technology in communication and computation.

Coherent destruction of tunneling (CDT) was discovered in a periodically-driving double well system [4]. It describes a fascinating phenomenon whereby coherent tunneling between two wells (or the Rabi oscillation between two states) is turned off by an externally enforced periodic level shift. Its understanding is related to dynamical localization [5], which occurs at isolated degenerate points of the quasienergies [4, 6]. CDT has thus far generated significant interest, and has been theoretically extended into various forms [7]–[18]. It has also been observed experimentally in many physical systems: including modulated optical coupler [19], driven double-well potentials for single-particle tunneling [20], and a single electron spin in diamond [21]. More recently, it has also found application in tuning the tunneling parameter of a Bose-Einstein condensate [22, 23].

Dark state is often discussed in terms of a three-state system where two of them are coupled coherently to the intermediate state, as in the model system we study here. When all coupling fields are on resonance with their respective coupled pair of states, we can adopt the rotating wave approximation and change into a suitable in-

teraction picture with all coupling coefficients becoming time independent. In this case, there always exists a dark state, whose eigenenergy becomes uniformly zero, and the corresponding eigenvector contains no projection onto the intermediate state. It is called dark as the intermediate state is an excited state capable of emitting photons. This type of dark state is also known as coherent population trapping [24], widely used in efficient population transfer through the stimulated Raman adiabatic passage (STIRAP) protocol. It has become the theoretical basis for several well-established implementations of quantum control and rudimentary quantum information processing gates.

In this Letter we report our surprising finding of an intimate relationship between dark state and coherent destruction of tunneling by studying a three-state system. In this system, two states are coherently coupled to an intermediate state and one of the two states shifts periodically against the other two. We find that CDT also exists in this three-state system, where the dynamical tunneling from one state to the other two is suppressed by the periodic driving over a range of parameters. However, this CDT for the three-state system has its own distinct feature: it is related to a dark Floquet state, which has zero quasi-energy and negligible population at the intermediate state. Quite interestingly, this dark Floquet state reduces to the well-known dark state in a non-driving three-state Λ -system [24] at high-frequency limit. Therefore, we call this CDT *dark coherent destruction of tunneling* (DCDT). These results can be generalized to N -state system. We also discuss a feasible experimental scheme where the visualization of the DCDT can be achieved readily.

Three-state system. The driving three-state system is described by the Schrödinger equation ($\hbar = 1$)

$$\begin{aligned} i\frac{dc_1}{dt} &= \frac{A}{2}\sin(\omega t)c_1 + vc_2, \\ i\frac{dc_2}{dt} &= -\frac{A}{2}\sin(\omega t)c_2 + vc_1 + vc_3, \end{aligned} \quad (1)$$

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$$i\frac{dc_3}{dt} = -\frac{A}{2}\sin(\omega t)c_3 + vc_2,$$

where c_1 , c_2 , and c_3 are the amplitudes at three states $|1\rangle$, $|2\rangle$, and $|3\rangle$, respectively. v is the coupling constant between the neighboring states. Energy state $|1\rangle$ is shifted periodically against the other two with driving strength A and frequency ω . The normalization condition $\sum_{j=1}^3 |c_j|^2 = 1$ is assumed.

To investigate the tunneling dynamics, we solve the time-dependent Schrödinger equation (1) numerically with initial state $(1, 0, 0)^T$. The evolution of the probability distribution $P_1 = |c_1|^2$ is presented in Fig. 1 for three typical driving conditions. For $A/\omega = 0$ (Fig. 1 (a)), we see that P_1 oscillates between zero and one, demonstrating no suppression of tunneling. For $A/\omega = 2.0$ (Fig. 1 (b)), the oscillations of P_1 are seen limited between 0.8 and 1, showing suppression of tunneling. At $A/\omega = 2.4$ (Fig. 1 (c)), P_1 remains near unity, signaling a complete suppression of tunneling between energy states. This is the quantum phenomenon well known as CDT [4].

We emphasize that what we find here is not a simple re-discovery of CDT in a three-state system. The CDT in this three-state model has its own distinct feature: the results shown in Fig. 1 (b,c) indicate that the suppression of tunneling occurs over a wide range of system parameters. This is in stark contrast to the CDT in a two-state system [4], which occurs only at isolated points of parameters. The widening of the suppression regime found in the driving three-state system is more clearly demonstrated in Fig. 1 (d), where the minimum value of P_1 is used to measure the suppression of tunneling. When $\min(P_1)$ is not zero, the tunneling is suppressed as the population is not allowed to be fully transferred from state $|1\rangle$ to the other two states. It is clear from Fig. 1 (d) that the suppression occurs as long as $A/\omega \neq 0$. For comparison, the results for the standard driven two-state system is plotted as dash dotted line in Fig. 1 (d), where the extremely narrow peak width indicates that CDT occurs only at isolated points of parameters.

There exists a fundamental reason why the CDT occurs at isolated system parameters in a two-state model, where the CDT is related to the degeneracy of quasi-energy levels [4] and the degeneracy usually happens only at isolated points. Therefore, the significant widening of the suppression parameter range that we see in Fig. 1 indicates that the CDT found here should have a different origin. To investigate this origin, we turn to the Floquet theory for a periodically-driving system. Similar to Bloch states for systems with spatially periodic potentials, the modulated system (1) has Floquet states, $(c_1, c_2, c_3)^T = (\tilde{c}_1, \tilde{c}_2, \tilde{c}_3)^T \exp(-i\varepsilon t)$, where ε is the quasi-energy and the amplitudes $(\tilde{c}_1, \tilde{c}_2, \tilde{c}_3)^T$ are periodic with modulation period $T = 2\pi/\omega$.

Our numerical results of the quasi-energies and Floquet states for the modulated system (1) are plotted in Fig. 2. There are three Floquet states with quasi-energies ε_1 , ε_2 , and ε_3 . We immediately notice from

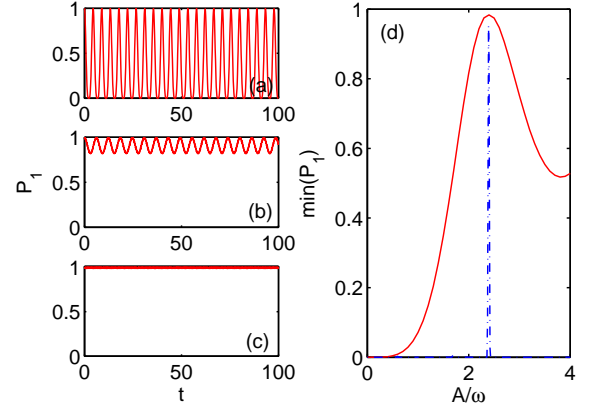


FIG. 1: (color online) The evolution of the probability at state $|1\rangle$ $P_1 = |c_1|^2$ for the system (1) for various driving conditions: (a) $A/\omega = 0$; (b) $A/\omega = 2.0$; (c) $A/\omega = 2.4$. (d) The minimum value of P_1 (solid line) as a function of driving parameter A/ω . The two-state results are plotted as a dash dotted line for comparison. The initial condition is $\{c_1 = 1, c_2 = 0, c_3 = 0\}$. The other parameters are $\omega = 10, v = 1$.

Fig. 2(a) that the quasi-energy ε_2 for the second Floquet state is always *zero* for all values of A/ω . We call this state *dark Floquet state* in analogy to the well-known dark state. This dark Floquet state stands out not only for its zero quasi-energy but also for its unique population distribution among energy states. We display the time-averaged population probability $\langle P_j \rangle = (\int_0^T dt |c_j|^2)/T$ for a given Floquet state $(c_1, c_2, c_3)^T$ in Fig. 2(b-d). The Floquet state with $\langle |P_j|^2 \rangle > 0.5$ is generally regarded as a state localized at the j -th energy state. As seen in Fig. 2(c), the dark Floquet state has almost zero population at energy state $|2\rangle$ while the population at $|1\rangle$ $\langle P_1 \rangle > 0.5$. In other words, the dark Floquet state is localized at $|1\rangle$. The other two Floquet states have identical population distribution. Since all their populations $\langle P_j \rangle \leq 0.5$, these two Floquet states are *not* localized.

It is not difficult to see the suppression of tunneling seen in Fig. 1 is linked to the existence of the dark Floquet state. We expand the initial state in terms of the Floquet states

$$(1, 0, 0)^T = b_1|\varepsilon_1\rangle + b_2|\varepsilon_2\rangle + b_3|\varepsilon_3\rangle. \quad (2)$$

During the dynamical evolution, the expansion coefficient b_i evolve as $b_i \exp(-i\varepsilon_i t)$. In other words, $|b_i|$ s are time independent. We look at the case $A/\omega = 2.4$, where $|\varepsilon_2\rangle$ has population one at state $|1\rangle$ while the other two states have zero population at $|1\rangle$. In this case, we have $|b_2| = 1$ and $b_1 = b_3 = 0$, which corresponds to a complete suppression of tunneling from $|1\rangle$ to $|2\rangle$ and $|3\rangle$. For other values of A/ω , similar arguments can be made. This shows that the CDT observed in Fig. 1 has a different origin: it is the consequence of dark Floquet states. Therefore, we call it *dark coherent destruction of tunneling* (DCDT).

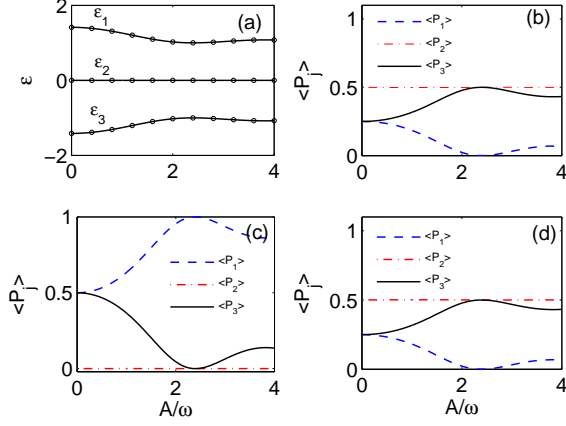


FIG. 2: (color online) (a) Quasienergies versus A/ω . Solid lines are for numerical results obtained from the original model (1) and circles are for the approximation results given by the effective model (3). Time-averaged populations for the Floquet state in the quasi-energy level (b) ϵ_1 , (c) ϵ_2 , and (d) ϵ_3 . The other parameters are $v = 1, \omega = 10$.

Interestingly, this dark Floquet state can be reduced to the well known dark state in a non-driven three-state Λ -system at high frequency limit. Introducing the transformation $c_m = a_m \exp[\pm i A \cos(\omega t)/(2\omega)]$ (+ for $m = 1$ and - for $m = 2, 3$) and averaging out high frequency terms, one can obtain a non-driven three-state system

$$\begin{aligned} i \frac{da_1}{dt} &= v J_0(A/\omega) a_2, \\ i \frac{da_2}{dt} &= v J_0(A/\omega) a_1 + v a_3, \\ i \frac{da_3}{dt} &= v a_2, \end{aligned} \quad (3)$$

where $J_0(A/\omega)$ is the zeroth order Bessel function. The famous dark state (also known as coherent trapped state) for Eq. (3) is given by $(a_1, a_2, a_3)^T = \frac{1}{\sqrt{\mathcal{M}}}(-v, 0, v J_0(A/\omega))^T$, where $\mathcal{M} = v^2 + [v J_0(A/\omega)]^2$. This dark state corresponds to the dark Floquet state. Similarly, this dark state is always localized at state $|1\rangle$ as $v > v J_0(A/\omega)$ and has zero population at state $|2\rangle$. This state is completely localized at state $|1\rangle$ when $J_0(A/\omega) = 0$. We have computed the eigenvalues of model (3) and compared them (circles) with the quasienergies (black solid lines) in Fig. 2 (a). The agreement is almost perfect.

Generalization to N -state system. Our analysis above is given for a three-state system and the original CDT was found in a two-state system. These results can be generalized to an N -state system, where one state is shifted periodically against all the other states. The equations of motion are

$$i \frac{dc_j}{dt} = v(c_{j-1} + c_{j+1}) + E_j(t)c_j, \quad (4)$$

$$E_1(t) = \frac{A}{2} \sin(\omega t), \quad E_{j \neq 1}(t) = -\frac{A}{2} \sin(\omega t),$$

where $c_{j \leq 0} = c_{j > N} = 0$.

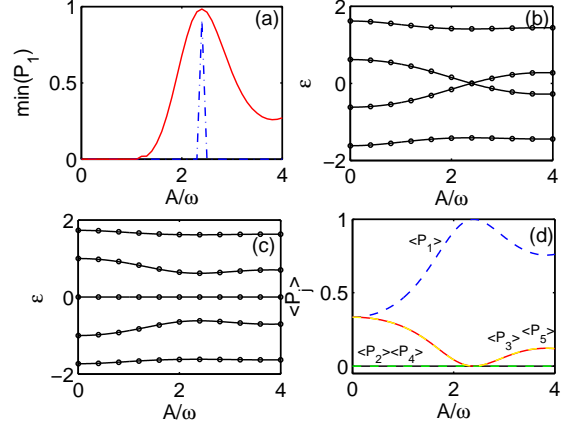


FIG. 3: (color online) (a) The minimum value of P_1 as a function of A/ω for $N = 4$ (dash dotted line) and $N = 5$ (solid line). The initial conditions are $c_1(0) = 1, c_j(0) = 0 (j \neq 1)$. Quasi-energies versus A/ω for (b) $N = 4$ and (c) $N = 5$. Solid lines are for numerical results obtained from the model (4) and circles are for the approximation results given by the effective model (5). (d) The time-averaged probability distribution of the Floquet state corresponding to $\epsilon = 0$ in Fig. 3 (c). The other parameters are $v = 1, \omega = 10$.

The quantum dynamics of the driven N -state systems is investigated by direct integration of the time-dependent Schrödinger equation (4) with the state initially prepared on the state $|1\rangle$. The CDT is found to exist. The minimum value of $P_1 = |c_1|^2$ as a function of A/ω is presented in Fig. 3 (a) for $N = 4$ and $N = 5$. When $N = 4$, the CDT occurs at an isolated point of parameters (dash dotted line in Fig. 3 (a)), where two of the four quasi-energy levels become degenerate (Fig. 3 (b)). This is exactly the same as in the two-state system. When $N = 5$, the parameter range where CDT occurs is extended substantially (solid line in Fig. 3 (a)) as in the three-state model. Furthermore, this five-state system also has a dark Floquet state: as seen in Fig. 3 (c), one of the quasi-energies always equals to zero. This dark Floquet state has negligible population at all of even j -th states (Fig. 3 (d)).

These numerical results with $N = 4, 5$, together with the known results for $N = 2, 3$, clearly suggest that (i) the dark state and the associated DCDT exist in odd N -state systems; (ii) the original CDT, which occurs at isolated parameter points, exists in all even- N -state system. This general conclusion can be proved analytically at high frequency limit.

Following the procedure used in the three-state system, we introduce the transformation $c_1 = a_1 \exp[-i \int A \sin(\omega t)/2 dt]$, $c_{j \neq 1} = a_j \exp[i \int A \sin(\omega t)/2 dt]$, where $a_j(z)$ are slowly varying functions. Using the expansion $\exp[\pm i A \cos(\omega t)/\omega] =$

$\sum_k (\pm i)^k J_k(A/\omega) \exp(\pm i k \omega t)$ in terms of Bessel functions and neglecting all orders except $k = 0$ for high frequency limit, we can reduce the coupled equations (4) to a non-driven model

$$i \frac{d\mathbf{a}}{dt} = \bar{H} \mathbf{a}, \quad (5)$$

where $\mathbf{a} = (a_1, a_2, \dots, a_N)^T$. The matrix \bar{H} is tridiagonal with $\bar{H}_{12} = \bar{H}_{21} = v_{\text{eff}} = v J_0(A/\omega)$, $\bar{H}_{n,n+1} = \bar{H}_{n+1,n} = v$. The effective coupling constant v_{eff} between state $|1\rangle$ and state $|2\rangle$ is tunable with the driving parameters.

The eigenvalues and eigenvectors of the tridiagonal $N \times N$ matrix \bar{H} enjoy some very interesting properties, whose rigorous proofs can be found in Supplemental Material.

(a) When N is even, for any nonzero v_{eff} and v , all the eigenvalues of the matrix \bar{H} are nonzero while two of the eigenvalues are zero for $v_{\text{eff}} = 0$.

Remark: This means that when N is even, two quasi-energy levels of the driven model (4) are degenerate at isolated points where $v_{\text{eff}} = v J_0(A/\omega) = 0$. The CDT occurs at these isolated points.

(b) When N is odd, one and only one eigenvalue of \bar{H} always equals to zero and, for the corresponding eigenvector $(w_1, w_2, \dots, w_N)^T$ of \bar{H} , the inequality $|w_1|^2 > 0.5$ holds for a finite range of parameters; for any other eigenvector $(w_1, w_2, \dots, w_N)^T$ of \bar{H} , one has $|w_j|^2 \leq 0.5$.

Remark: When N is odd, the system always has one and only one dark Floquet state, which is localized over a finite range of parameters. Correspondingly, DCDT occurs over a finite range of parameters.

Experimental observation. By mapping the temporal evolution of quantum systems into the spatial propagations of light waves, the engineered waveguides have provided an ideal platform to investigate a wide variety of coherent quantum effects[25, 26]. The phenomenon of DCDT can also be observed with this kind of waveguide system. The discrete model (4) can be simulated by the light propagation in an array of N waveguides placed closely and with equal spacing. Periodic driving is realized by the harmonic modulation of the refractive index of the waveguides along the propagation direction[27, 28]. For our system, the periodic modulation of the first waveguide has a phase difference of π against the modulations for all other $N - 1$ waveguides. When N is odd, the DCDT can be readily observed with current experimental capacity[27, 28].

In summary, we find that the CDT also happens in a three-state quantum system, where one energy state is shifted periodically against the other two states. We call this type of CDT dark coherent destruction of tunneling (DCDT) as it is related to the existence of a dark Floquet state in the three-state system. The dark Floquet state has zero quasi-energy and negligible population at the intermediate state. It reduces to the well known dark state of a non-driven three-state system. These results can be generalized to a periodically driven N -state system. We have also pointed out that observation of DCDT is well within the capacity of current experiments.

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Supplemental Material

The $N \times N$ tridiagonal matrix \bar{H} has the following non-zero matrix elements: $\bar{H}_{12} = \bar{H}_{21} = v_{\text{eff}}$, $\bar{H}_{n,n+1} = \bar{H}_{n+1,n} = v$ for $n = 1, 2, \dots, N - 1$. v_{eff} is tunable and $v \neq 0$ is fixed. The eigenvalues and eigenvectors of \bar{H} have the following properties.

Property 1. When N is odd, one and only one eigenvalue of \bar{H} always equals to zero.

Proof. Let $\lambda_1, \lambda_2, \dots, \lambda_N$ be all the eigenvalues of matrix \bar{H} , then $D_N = \det(\bar{H}) = \lambda_1 \lambda_2 \cdots \lambda_N$. It is easy to verify that $D_2 = -v_{\text{eff}}^2$, $D_1 = D_3 = 0$, and the relation $D_N = -v^2 D_{N-2}$ ($N \geq 3$). Therefore, one has $D_{2k-1} = 0$ and $D_{2k} = (-1)^k v^{2k-2} v_{\text{eff}}^2$ ($k = 1, 2, 3, \dots$). When N is odd, $D_N = \lambda_1 \lambda_2 \cdots \lambda_N = 0$, which means that at least one eigenvalue equals to zero regardless the values of v_{eff} and v . For the zero eigenvalue, the eigen-equation is $\bar{H} \mathbf{w} = 0$, where $\mathbf{w} = (w_1, w_2, \dots, w_N)^T$. When expanded, the equation is turned into the following equations: $v_{\text{eff}} w_2 = 0$, $v_{\text{eff}} w_1 + v w_3 = 0$, $v w_{j-1} + v w_{j+1} = 0$ ($j = 3, 4, \dots, N - 1$), $v w_{N-1} = 0$. There is only one non-trivial solution. For $v_{\text{eff}} \neq 0$, it is given by

$$\mathbf{w} = \frac{1}{\sqrt{\mathcal{M}}} (w_1^0, w_2^0, \dots, w_N^0)^T \quad (6)$$

with $w_1^0 = (-1)^{(N-1)/2} v / v_{\text{eff}}$, $w_{2k}^0 = 0$, and $w_{2k+1}^0 = (-1)^{(N-2k-1)/2}$ where $k = 1, 2, \dots, (N - 1)/2$ and $\mathcal{M} = \sum_{j=1}^N |w_j^0|^2$ is the normalization factor. For $v_{\text{eff}} = 0$, the solution is $\mathbf{w} = (1, 0, \dots, 0)^T$. This shows matrix \bar{H} has one and only one eigenvalue equal to zero. \square

Property 2. When N is even, for any nonzero v_{eff} , all the eigenvalues of the matrix \bar{H} are nonzero while two of the eigenvalues are zero for $v_{\text{eff}} = 0$.

Proof. If $v_{\text{eff}} \neq 0$, when N is even, then $D_N = (-1)^{N/2} v^{N-2} v_{\text{eff}}^2 = \lambda_1 \lambda_2 \cdots \lambda_N \neq 0$. Thus all the eigenvalues of the matrix \bar{H} are nonzero. When $v_{\text{eff}} = 0$, it is obvious that $D_N = \lambda_1 \lambda_2 \cdots \lambda_N = 0$. There are zero eigenvalues. With $v_{\text{eff}} = 0$, the tridiagonal matrix of \bar{H}

is divided into two uncoupled subspaces, i.e., $\bar{H} = 0 \oplus F$, where F is a tridiagonal matrix with nonzero elements $F_{n,n+1} = F_{n+1,n} = v$. It is clear that F possesses Property 1 and has only one zero eigenvalue. \bar{H} thus has two zero eigenvalues. \square

Property 3. *If λ is an eigenvalue of \bar{H} with eigenvector $(w_1, w_2, \dots, w_N)^T$, then $-\lambda$ is an eigenvalue of \bar{H} with the corresponding eigenvector $(w'_1, w'_2, \dots, w'_N)^T$ where $w'_j = (-1)^j w_j$.*

Proof. The eigenvalue equation $\bar{H}\mathbf{w} = \lambda\mathbf{w}$ can be written in the form $\sum_{j=1}^N \bar{H}_{ij} w_j = \lambda w_i$, where $\bar{H}_{ij} = 0$ when $|i - j| = 0$ and $|i - j| \geq 2$. Multiplying by $(-1)^{i-1} \lambda$, we obtain $\sum_{j=1}^N \bar{H}_{ij} (-1)^j w_j = -\lambda (-1)^i w_i$ and have the proof. \square

Property 4. *When N is odd, for the eigenvector*

$(w_1, w_2, \dots, w_N)^T$ of \bar{H} belonging to $\lambda = 0$, the inequality $|w_1|^2 > 1/2$ holds for a finite range of parameters; For an eigenvector $(w_1, w_2, \dots, w_N)^T$ of \bar{H} belonging to $\lambda \neq 0$, one has that $|w_j|^2 \leq 1/2$, whether N is odd or even.

Proof. According to Eq.(6), it is clear that the inequality $|w_1|^2 > 0.5$ is valid only when $(v/v_{\text{eff}})^2 > (N - 1)/2$. When $\lambda \neq 0$, the two eigenvalues λ and $-\lambda$ are distinct and the corresponding eigenvectors are orthogonal to each other. With Property 3, one has $\sum_{k=1}^{(N+1)/2} |w_{2k-1}|^2 = \sum_{k=1}^{(N-1)/2} |w_{2k}|^2$ when N is odd; $\sum_{k=1}^{N/2} |w_{2k-1}|^2 = \sum_{k=1}^{N/2} |w_{2k}|^2$ when N is even. With the normalization condition $\sum_{j=1}^N |w_j|^2 = 1$, we immediately obtain that $|w_j|^2 \leq 0.5$. \square

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